Eigenschaften der Laplacetransformation

Ähnlichkeit ($a > 0$).

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "t \mapsto at"'] & F(s) \arrow[d] \\ f(at) \arrow[r, "\mathcal{L}"'] & \frac{1}{a} F\left(\frac{s}{a}\right) \end{tikzcd}$$

Dämpfung.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\cdot e^{-at}"'] & F(s) \arrow[d, "s \mapsto s+a"] \\ e^{-at} f(t) \arrow[r, "\mathcal{L}"'] & F(s+a) \end{tikzcd}$$

Verschiebung ($a > 0$).

$$\begin{tikzcd} H(t)f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "t \mapsto t-a"'] & F(s) \arrow[d, "\cdot e^{-as}"] \\ H(t-a)f(t-a) \arrow[r, "\mathcal{L}"'] & e^{-as} F(s) \end{tikzcd}$$

Ableitung Zeitbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\frac{d}{dt}"'] & F(s) \arrow[d] \\ f'(t) \arrow[r, "\mathcal{L}"'] & sF(s) - f(0^+) \end{tikzcd}$$

Ableitung Bildbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\cdot(-t)"'] & F(s) \arrow[d, "\frac{d}{ds}"] \\ -tf(t) \arrow[r, "\mathcal{L}"'] & F'(s) \end{tikzcd}$$

Integral Zeitbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d] & F(s) \arrow[d, "\cdot \frac{1}{s}"] \\ \int_0^t f(u)du \arrow[r, "\mathcal{L}"'] & \frac{1}{s} F(s) \end{tikzcd}$$

Integral Bildbereich.

$$\begin{tikzcd} f(t) \arrow[r, "\mathcal{L}"] \arrow[d, "\cdot \frac{1}{t}"'] & F(s) \arrow[d] \\ \frac{1}{t} f(t) \arrow[r, "\mathcal{L}"'] & \int_s^{+\infty} F(u)du \end{tikzcd}$$

Faltung.

$$\begin{tikzcd} H(t)f_1(t) \arrow[r] \arrow[d, "\mathcal{L}"'] & (f_1 * f_2)(t) \arrow[d, "\mathcal{L}"'] & H(t)f_2(t) \arrow[l] \arrow[d, "\mathcal{L}"'] \\ F_1(s) \arrow[r] & F_1(s) \cdot F_2(s) & F_2(s) \arrow[l] \end{tikzcd}$$

$$(f_1 * f_2)(t) = \int_0^t f_1(u) f_2(t-u)du, \quad \text{für} \quad t \geq 0.$$

Transformationen

$f(t)$	$F(s) = \mathcal{L}(f(t))$
t^n	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin(t)$	$\dfrac{1}{s^2+1}$
$\cos(t)$	$\dfrac{s}{s^2+1}$
$\delta(t)$	1
$f(t+T) = f(t)$	$\frac{1}{1-e^{-sT}} \int_0^T f(t) e^{-st} dt$