Fourierreihe

Eulerformeln

$$e^{jx} = \cos(x) + j\sin(x),$$
$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}, \quad \cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$
$$T = \frac{2\pi}{\omega} \qquad \omega = 2\pi\nu$$

Fourierreihe von T-periodischen Funktionen

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \Big(a_n \cos(n\omega t) + b_n \sin(n\omega t)\Big)$$
$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt \qquad a_0 = \frac{2}{T}\int_0^T f(t)dt$$
$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt$$

Komplexe Darstellung

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}, \qquad c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega t}dt$$

Umrechnung:

$$c_0 = \frac{a_0}{2}, \qquad c_n = \frac{1}{2}(a_n - jb_n),$$
$$c_{-n} = \frac{1}{2}(a_n + jb_n), \qquad n = 1, 2, 3, \ldots$$

Satz von Parseval

$$\frac{1}{T}\int_0^T f(t)^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

Fouriertransformation

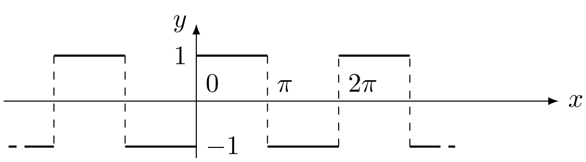
$$\hat{f}(s) = \int_{-\infty}^{\infty} f(t)e^{-j2\pi st}dt, \qquad \check{g}(t) = \int_{-\infty}^{\infty} g(s)e^{j2\pi st}ds$$

Sinc-Funktion

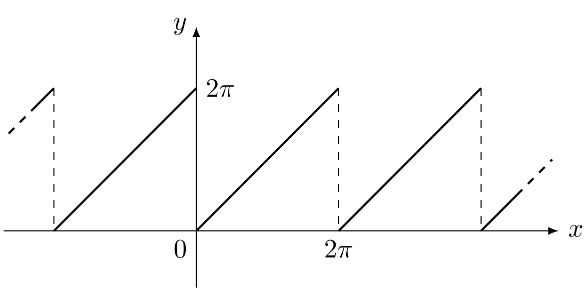
$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Tabellen

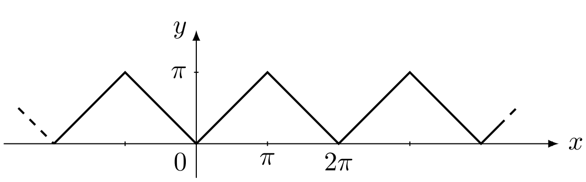
Wichtige Fourierreihen



$$f(x) = \frac{4}{\pi}\sum_{n \text{ ung.}} \frac{\sin(nx)}{n}$$



$$f(x) = \pi - 2\sum_{n=1}^{\infty} \frac{\sin(nx)}{n}$$



$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\sum_{n \text{ ung.}} \frac{\cos(nx)}{n^2}$$